

# Constraints of unparticle physics parameters from $K^0 - \bar{K}^0$ mixing

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## Abstract

Kaon mixing plays an important role in test of the Standard Model (SM) and new physics beyond it. Scale invariant unparticle physics leads to new effects on the meson-antimeson oscillation at the tree level. In this study, we investigate the unparticle effects on  $K^0 - \bar{K}^0$  mixing. Base on the current experimental data, we give constraints of  $K^0 - \bar{K}^0$  mixing on unparticle parameters.

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## I. INTRODUCTION

K meson had played important role in parity violation [1], and CP violation was first discovered in the natural meson [2]. Up to now, the indirect CP violation is unambiguously and precisely measured in FCNC process of  $K^0 - \bar{K}^0$  mixing, the parameter  $\varepsilon_K$ , which describes the CP violation in  $K^0 - \bar{K}^0$  mixing, we can test the standard model together with the unitarity of  $V_{CKM}$ , from which some parameters of the Standard Model will be constrained.  $K^0 - \bar{K}^0$  mixing is valued very much in finding new physics, such as, one can use direct and indirect kaon CP violation to constrain the Warped kk scale[3], yukawa unification may be probed with  $K^0 - \bar{K}^0$  mixing [4], Kaon mixing will give additional constraints on right-handed scale in super symmetry left-right symmetric mode [5], kaon mixing in wrapped extra dimensions with custodial protection has also been studied [6].

Georgi observed that unparticle with scale dimension looks like a non-integral number  $d_U$  of invisible massless particles[9]. We know that scale invariance is the property of classical field theory. Any transformation of variables must observe the rule of scale invariance. It is still very important for the research of the high energy physics. For an asymptotically free theory, the scale invariance is recovered at high energy limit, for QCD as an example. With the renormalization group method, the breaking of scale invariance will be included in the anomalous dimensions of operators here[7]. Stuff with scale invariance in the infrared region would be so different and less known to us[8]. However we will not give up it for its simple and attractive. We consider a scale invariant sector of an effective field theory that fulfil the special conditions.

Recently, in the unparticle domain, Meson-antimeson oscillation is study by many articles, such as  $K^0 - \bar{K}^0$ ,  $D^0 - \bar{D}^0$ ,  $B_d^0 - \bar{B}_d^0$  and  $B_s^0 - \bar{B}_s^0$  of the neutral meson systems. Therein,  $D^0 - \bar{D}^0$ ,  $B_d^0 - \bar{B}_d^0$  and  $B_s^0 - \bar{B}_s^0$  is particularly researched by some articles [10] [12] [13], but only  $K^0 - \bar{K}^0$  is not clearly known by us. Because at Long distance contributions to oscillation parameters for  $K^0 - \bar{K}^0$  are larger, so we still not know its characteristic clearly. This article will try to do some research on the  $K^0 - \bar{K}^0$  mixing. Long distance effects contributes about 30% in  $\Delta M_K$  domains the  $\Delta\Gamma_K$ , we will attribute them to unparticle effects.

## II. $K^0 - \bar{K}^0$ MIXING IN THE SM

At first, we give notations for the neutral kaon system. In the  $K^0 - \bar{K}^0$  system, the oscillation between the two neutral kaon mesons is described by the equation

$$i\frac{d\psi(t)}{dt} = \hat{H}\psi(t), \quad \psi(t) = \begin{pmatrix} |K^0(t)\rangle \\ |\bar{K}^0(t)\rangle \end{pmatrix}, \quad (1)$$

where

$$\hat{H} = \hat{M} - i\frac{\hat{\Gamma}}{2} = \begin{pmatrix} M - i\frac{\Gamma}{2} & M_{12} - i\frac{\Gamma_{12}}{2} \\ M_{12}^* - i\frac{\Gamma_{12}^*}{2} & M - i\frac{\Gamma}{2} \end{pmatrix}. \quad (2)$$

with  $M_{ij}$  and  $\Gamma_{ij}$  being the transition matrix elements. We have assumed the CPT conservation and hermiticity for the matrices  $\hat{M}$  and  $\hat{\Gamma}$ .

After diagonalizing the system and using the convention  $CP|K^0(t)\rangle = -|\bar{K}^0(t)\rangle$ ,  $CP|\bar{K}^0(t)\rangle = -|K^0(t)\rangle$ , we obtain the eigenstates:

$$K_{L,S} = \frac{(1 + \bar{\varepsilon})K^0 \pm (1 - \bar{\varepsilon})\bar{K}^0}{\sqrt{2(1 + |\bar{\varepsilon}|^2)}}, \quad (3)$$

where  $\bar{\varepsilon}$  is a small complex parameter given by

$$\frac{1 - \bar{\varepsilon}}{1 + \bar{\varepsilon}} = \sqrt{\frac{M_{12}^* - i\frac{1}{2}\Gamma_{12}^*}{M_{12} - i\frac{1}{2}\Gamma_{12}}}. \quad (4)$$

In the SM, the oscillation between the flavor states  $K^0$  and  $\bar{K}^0$  are caused by weak interactions, thus the above eigenstates are called by the weak eigenstates. In new physics which is beyond the SM, the eigenstates represent generalized states including both the SM and new physics effects.

The eigenvalues associated with the eigenstates are

$$M_{L,S} = M \pm \text{Re}Q, \quad \Gamma_{L,S} = \Gamma \mp 2\text{Im}Q, \quad (5)$$

where

$$Q = \sqrt{\left(M_{12} - i\frac{1}{2}\Gamma_{12}\right)\left(M_{12}^* - i\frac{1}{2}\Gamma_{12}^*\right)}, \quad (6)$$

Consequently we get

$$\Delta M = M(K_L) - M(K_S) = 2\text{Re}Q, \quad \Delta\Gamma = \Gamma(K_L) - \Gamma(K_S) = -4\text{Im}Q. \quad (7)$$

Since  $\bar{\varepsilon}$  is small, at the order of  $\mathcal{O}(10^{-3})$ , the below relation is reasonable,

$$\text{Im}M_{12} \ll \text{Re}M_{12}, \quad \text{Im}\Gamma_{12} \ll \text{Re}\Gamma_{12}. \quad (8)$$

It should be noted that the above relation is still valid in the unparticle physics. Thus, neglecting the small imaginary part of  $M_{12}$  and  $\Gamma_{12}$ , we can get a good approximation:

$$\Delta M_K \cong 2\text{Re}M_{12}, \quad \Delta\Gamma_K \cong 2\text{Re}\Gamma_{12}. \quad (9)$$

For the mixing parameter  $\bar{\varepsilon}$ , it depends on the phase convention. A convenient choice is Wu-Yang convention with  $\text{Im}A_0 = 0$ . Under this phase convention, an important formula for  $\bar{\varepsilon}$  is [11]

$$\bar{\varepsilon} = \frac{i}{1+i} \frac{\text{Im}M_{12}}{\Delta M_K}. \quad (10)$$

Now, we discuss the  $K^0 - \bar{K}^0$  mixing in the SM. At the quark level, the flavor changing neutral current transitions between  $K^0$  and  $\bar{K}^0$  are induced through a box diagram with exchange of the intermediate up type quarks. From [16], the SM contribution to  $M_{12}$  is

$$M_{12}^{\text{SM}} = \frac{G_F^2}{12\pi^2} f_K^2 \hat{B}_K m_K m_W^2 \left[ \lambda_c^{*2} \eta_1 S_0(x_c) + \lambda_t^{*2} \eta_2 S_0(x_t) + 2\lambda_c^* \lambda_t^* \eta_3 S_0(x_c, x_t) \right], \quad (11)$$

where  $f_K$  is the  $K$ -meson decay constant,  $m_K$  the  $K$ -meson mass,  $\hat{B}_K$  the renormalization group invariant parameter. The parameters  $\lambda_i = V_{is}^* V_{id}$  with  $V_{is}$  and  $V_{id}$  the CKM matrix elements. The functions  $S_0$  are

$$\begin{aligned} S_0(x_t) &= \frac{4x_t - 11x_t^2 + x_t^3}{4(1-x_t)^2} - \frac{3x_t^3 \ln x_t}{2(1-x_t)^3}, \\ S_0(x_c) &= x_c, \\ S_0(x_c, x_t) &= x_c \left[ \ln \frac{x_t}{x_c} - \frac{3x_t}{4(1-x_t)} - \frac{3x_t^2 \ln x_t}{4(1-x_t)^2} \right], \end{aligned} \quad (12)$$

The  $\eta_i$  are taken to be next-to-leading-order (NLO) values given in [? ? ? ? ]

$$\eta_1 = 1.38 \pm 0.20, \quad \eta_2 = 0.57 \pm 0.01, \quad \eta_3 = 0.47 \pm 0.04. \quad (13)$$

For the  $\Gamma_{12}$ , the short-distance contribution arises from the on-shell quark interactions which is expected to be very small. The long-distance contribution coming from the intermediate hadron states would dominate. However, the precise calculation of this part suffers from the non-perturbative uncertainties. From the new physics point of view, it is difficult to disentangle the non-perturbative uncertainties from the new physics contribution. In this study, we make an assumption that  $\Gamma_{12}$  is dominated by new physics, in particular the unparticle, and the SM contribution is  $\Gamma_{12}^{\text{SM}} \approx 0$ . This approximation can be improved when the long-distance SM contribution is included.

In the above derivations and the forthcoming discussions in the unparticle physics, the below relations are useful

$$\begin{aligned} \langle \bar{K}^0 | \bar{s} \gamma_\mu (1 - \gamma_5) d \bar{s} \gamma^\mu (1 - \gamma_5) d | K^0 \rangle &= \frac{8}{3} f_K^2 m_K^2 \hat{B}_K, \\ \langle \bar{K}^0 | \bar{s} (1 - \gamma_5) d \bar{s} (1 - \gamma_5) d | K^0 \rangle &= -\frac{5}{3} f_K^2 m_K^2 \hat{B}_K \left( \frac{m_K}{m_s + m_d} \right)^2. \end{aligned} \quad (14)$$

where  $m_{s,d}$  are the strange and down quark masses.

### III. $K^0 - \bar{K}^0$ MIXING IN UNPARTICLE PHYSICS

In this section, we will study the  $K^0 - \bar{K}^0$  mixing from unparticle physics. In the low energy effective theory, unparticle fields will emerge below an energy scale  $\Lambda_U$  [9]. The relevant low energy effective interactions for the  $s$  and  $d$  quarks is

$$\frac{C_S^{ds}}{\Lambda_U^{d_U}} \bar{s} \gamma_\mu (1 - \gamma_5) d \partial^\mu O_U + \frac{C_V^{ds}}{\Lambda_U^{d_U-1}} \bar{s} \gamma_\mu (1 - \gamma_5) d O_U^\mu + h.c. . \quad (15)$$

where  $O_U$  and  $O_U^\mu$  denote operators of the scalar and vector unparticle fields respectively. The  $C_S$  and  $C_V$  are dimensionless coefficients and they depend on quark and lepton flavors in general. Since only quarks  $s$  and  $d$  are concerned in this study, we will simplify  $C_S^{ds} \rightarrow C_S$  and  $C_V^{ds} \rightarrow C_V$  in the later discussions.

The propagators for the scalar and vector unparticle fields with the time-like momentum  $P$  are given by [14, 15]

$$\begin{aligned} \int d^4x e^{iP \cdot x} \langle 0 | T O_U(x) O_U(0) | 0 \rangle &= i \frac{A_{d_U}}{2 \sin(d_U \pi)} \frac{1}{(P^2 + i\epsilon)^{2-d_U}} e^{-id_U \pi}, \\ \int d^4x e^{iP \cdot x} \langle 0 | T O_U^\mu(x) O_U^\nu(0) | 0 \rangle &= i \frac{A_{d_U}}{2 \sin(d_U \pi)} \frac{-g^{\mu\nu} + P^\mu P^\nu / P^2}{(P^2 + i\epsilon)^{2-d_U}} e^{-id_U \pi}, \end{aligned} \quad (16)$$

where

$$A_{d_U} = \frac{16\pi^{5/2}}{(2\pi)^{2d_U}} \frac{\Gamma(d_U + 1/2)}{\Gamma(d_U - 1)\Gamma(2d_U)}. \quad (17)$$

We have assumed that the vector unparticle is transverse:  $\partial_\mu O_U^\mu = 0$ . The scale dimension  $d_U$  is fractional in general, and it cannot be integral (except  $d_U = 1$  where the singularity is canceled by  $A_{d_U}$  in the nominator) due to the singularity of the function  $\sin(d_U \pi)$  in the denominator. The factor  $e^{-id_U \pi}$  provides a CP conserving phase which produces peculiar interference effects in high energy scattering processes [14], Drell-Yan process [15] and CP violation in B decays [12].

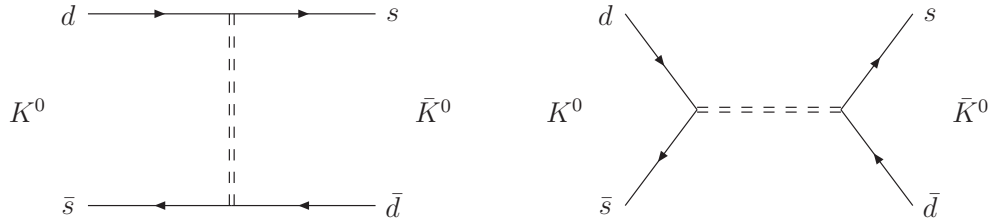


FIG. 1: The Feynman graphs of the  $K^0 - \bar{K}^0$  mixing in unparticle physics. The double dashed lines represent the unparticle fields.

Unlike the SM where the  $\Delta S = 2$  FCNC transitions occurred at loop level, the unparticle contribution can induce a tree-level FCNC transitions between  $K^0$  and  $\bar{K}^0$  which is depicted

in Fig. 1. There are two diagrams corresponding to t- and s-channel unparticle exchanges. For the s-channel, the momentum of the unparticle  $P^2 = m_K^2$ . For the t-channel, the momentum of the unparticle is not fixed which introduces an uncertainty in the calculation. Because strange quark mass is much greater than that of the down quark,  $m_s \gg m_d$ , one may think that the momentum of the kaon meson is mostly carried by the strange quark. The momentum of the exchanged unparticle will be at the order of  $m_K$ , and we make an approximation that  $P^2 \approx m_K^2$ .

Using the Feynman rules given above, we obtain the final expression for the transition matrices in the unparticle physics. For the scalar unparticle,

$$\begin{aligned} M_{12}^{\mathcal{U}} &= \frac{5C_S^2}{12} \frac{f_K^2 \hat{B}_K A_{d\mathcal{U}}}{m_K} \left( \frac{m_K}{\Lambda_{\mathcal{U}}} \right)^{2d_{\mathcal{U}}} \left( \frac{m_K}{m_s + m_d} \right)^2 \cot(d_{\mathcal{U}}\pi), \\ \Gamma_{12}^{\mathcal{U}} &= -\frac{5C_S^2}{6} \frac{f_K^2 \hat{B}_K A_{d\mathcal{U}}}{m_K} \left( \frac{m_K}{\Lambda_{\mathcal{U}}} \right)^{2d_{\mathcal{U}}} \left( \frac{m_K}{m_s + m_d} \right)^2, \end{aligned} \quad (18)$$

and

$$\begin{aligned} M_{12}^{\mathcal{U}} &= \frac{C_V^2}{4} \frac{f_K^2 \hat{B}_K A_{d\mathcal{U}}}{m_K} \left( \frac{m_K}{\Lambda_u} \right)^{2d_u-2} \left[ \frac{8}{3} - \frac{5}{3} \left( \frac{m_K}{m_s + m_d} \right)^2 \right] \cot(d_{\mathcal{U}}\pi), \\ \Gamma_{12}^{\mathcal{U}} &= -\frac{C_V^2}{2} \frac{f_K^2 \hat{B}_K A_{d\mathcal{U}}}{m_K} \left( \frac{m_K}{\Lambda_u} \right)^{2d_u-2} \left[ \frac{8}{3} - \frac{5}{3} \left( \frac{m_K}{m_s + m_d} \right)^2 \right]. \end{aligned} \quad (19)$$

for the vector unparticle. For both cases, we have

$$M_{12}^{\mathcal{U}} = \Gamma_{12}^{\mathcal{U}} \cot(d_{\mathcal{U}}\pi). \quad (20)$$

The above relation has been found in [17].

For the mixing parameter  $\bar{\varepsilon}$ , it is straightforwardly to obtain

$$\bar{\varepsilon}^{\mathcal{U}} = \frac{i}{1+i} \frac{\text{Im} M_{12}^{\mathcal{U}}}{\Delta M_K^{\text{exp}}}. \quad (21)$$

where we have used the  $\Delta M_K^{\text{exp}}$  to replace the  $\Delta M_K$  in Eq. (10). The above formula is applicable for both the scalar and vector unparticles.

#### IV. CONSTRAINTS OF THE UNPARTICLE PARAMETERS

In the  $K^0 - \bar{K}^0$  system, the mass difference  $\Delta M_K$ , width difference  $\Delta \Gamma_K$  and the CP violating parameter  $\bar{\varepsilon}$  are the most important experimental parameters. We will make use of these data to constrain the phenomenological parameters of the unparticle physics. From PDG [19], the experimental results are

$$\begin{aligned} \Delta M_K^{\text{exp}} &= 3.483 \times 10^{-15} \text{ GeV}, \\ \Delta \Gamma_K^{\text{exp}} &= -7.348 \times 10^{-15} \text{ GeV}, \\ |\bar{\varepsilon}|^{\text{exp}} &= 2.228 \times 10^{-3}. \end{aligned} \quad (22)$$

Here only the central values are given. In the previous sections, we have obtained the relations between the experiment and theory as

$$\begin{aligned}\Delta M_K^{\text{exp}} &= 2\text{Re}(M_{12}^{\text{SM}} + M_{12}^{\mathcal{U}}), \\ \Delta \Gamma_K^{\text{exp}} &= 2\text{Re}\Gamma_{12}^{\mathcal{U}}, \\ |\bar{\varepsilon}|^{\text{exp}} &= \frac{\text{Im}M_{12}^{\text{SM}} + \text{Im}M_{12}^{\mathcal{U}}}{\sqrt{2}\Delta M_K^{\text{exp}}}.\end{aligned}\tag{23}$$

In the last equation, we have approximated  $\bar{\varepsilon}$  by  $\bar{\varepsilon} \approx |\bar{\varepsilon}|e^{i\frac{\pi}{4}}$ . The SM contribution of  $M_{12}^{\text{SM}}$  is given in Eq. (11) and can be calculated reliably. The main uncertainty in SM comes from the parameter  $\hat{B}_K$ . If there is no QCD corrections, it is 1. The non-perturbative effects are estimated to be of order 30% [18]. In this study, we will use the value  $\hat{B}_K = 0.75 \pm 0.15$  as in [18].

After subtracting the SM contribution, the remained is the unparticle effect. Then we can know  $\text{Re}M_{12}^{\mathcal{U}}$ ,  $\text{Re}\Gamma_{12}^{\mathcal{U}}$  and  $\text{Im}M_{12}^{\mathcal{U}}$ . Using Eqs. (18, 19, 21), we obtain the constraints of  $C_S$ ,  $C_V$  with  $d_{\mathcal{U}}$  as following. From  $\text{Re}M_{12}^{\mathcal{U}}$ ,  $\text{Re}C_S^2$  and  $\text{Re}C_V^2$  are obtained as

$$\begin{aligned}\text{Re}C_S^2 &= \frac{12(m_s + m_d)^2}{5A_{d_{\mathcal{U}}}\hat{B}_K f_K^2 m_K} \left(\frac{m_K}{\Lambda_{\mathcal{U}}}\right)^{-2d_{\mathcal{U}}} \tan(d_{\mathcal{U}}\pi) \text{Re}M_{12}^{\mathcal{U}}, \\ \text{Re}C_V^2 &= \frac{4m_K}{A_{d_{\mathcal{U}}}\hat{B}_K f_K^2 \left[\frac{8}{3} - \frac{5}{3}\left(\frac{m_K}{m_s+m_d}\right)^2\right]} \left(\frac{m_K}{\Lambda_{\mathcal{U}}}\right)^{2-2d_{\mathcal{U}}} \tan(d_{\mathcal{U}}\pi) \text{Re}M_{12}^{\mathcal{U}}.\end{aligned}\tag{24}$$

From  $\text{Re}\Gamma_{12}^{\mathcal{U}}$ ,  $\text{Re}C_S^2$  and  $\text{Re}C_V^2$  are obtained as

$$\begin{aligned}\text{Re}C_S^2 &= \frac{-6(m_s + m_d)^2}{5A_{d_{\mathcal{U}}}\hat{B}_K f_K^2 m_K} \left(\frac{m_K}{\Lambda_{\mathcal{U}}}\right)^{-2d_{\mathcal{U}}} \text{Re}\Gamma_{12}^{\mathcal{U}}, \\ \text{Re}C_V^2 &= \frac{-2m_K}{A_{d_{\mathcal{U}}}\hat{B}_K f_K^2 \left[\frac{8}{3} - \frac{5}{3}\left(\frac{m_K}{m_s+m_d}\right)^2\right]} \left(\frac{m_K}{\Lambda_{\mathcal{U}}}\right)^{2-2d_{\mathcal{U}}} \text{Re}\Gamma_{12}^{\mathcal{U}},\end{aligned}\tag{25}$$

From  $\text{Im}\Gamma_{12}^{\mathcal{U}}$ ,  $\text{Im}C_S^2$  and  $\text{Im}C_V^2$  are obtained as

$$\begin{aligned}\text{Im}C_S^2 &= \frac{12(m_s + m_d)^2}{5A_{d_{\mathcal{U}}}\hat{B}_K f_K^2 m_K} \left(\frac{m_K}{\Lambda_{\mathcal{U}}}\right)^{-2d_{\mathcal{U}}} \tan(d_{\mathcal{U}}\pi) \text{Im}M_{12}^{\mathcal{U}}, \\ \text{Im}C_V^2 &= \frac{4m_K}{A_{d_{\mathcal{U}}}\hat{B}_K f_K^2 \left[\frac{8}{3} - \frac{5}{3}\left(\frac{m_K}{m_s+m_d}\right)^2\right]} \left(\frac{m_K}{\Lambda_{\mathcal{U}}}\right)^{2-2d_{\mathcal{U}}} \tan(d_{\mathcal{U}}\pi) \text{Im}M_{12}^{\mathcal{U}}.\end{aligned}\tag{26}$$

### A. Numerical analysis

At first, we collect all the input parameters here[19].

CKM parameters ( $A$ ,  $\lambda$ ,  $\bar{\rho}$ ,  $\bar{\eta}$ ):

$$A = 0.804_{-0.015}^{+0.022}, \quad \lambda = 0.2253 \pm 0.0007, \quad \bar{\rho} = 0.132_{-0.014}^{+0.022}, \quad \bar{\eta} = 0.341 \pm 0.013.\tag{27}$$

Decay constant of kaon meson:

$$f_K = 160 \text{ MeV}. \quad (28)$$

Quark and gauge boson masses:

$$\begin{aligned} m_d &= 4.1 - 5.8 \text{ MeV}, & m_s &= 101_{-21}^{29} \text{ MeV}, & m_c &= 1.27_{-0.09}^{+0.07} \text{ GeV}, \\ m_t &= 171.2 \pm 0.9 \pm 1.3 \text{ GeV}, & m_W &= 80.384 \pm 0.014 \text{ GeV} \end{aligned} \quad (29)$$

## B. SM results

$$\begin{aligned} M_{12} &= 1.07841 \times 10^{-15} - 1.0445 \times 10^{-17}i \text{ GeV} \\ \Delta M_K^{SM} &= 2.157 \text{ GeV} \\ |\bar{\varepsilon}|^{SM} &= 2.120 \times 10^{-3} \end{aligned} \quad (30)$$

## C. Unparticle

The numerical results are given in Table I. This quantitative data tables is evidence to give feedback of the the constrain of the dimensionless coefficients  $C_S$ ,  $C_V$  and fraction dimension " $d_U$ ".

## D. $\sqrt{\text{Re}C_S^2}$ and $\sqrt{\text{Re}C_V^2}$ , $\sqrt{\text{Im}C_S^2}$ and $\sqrt{\text{Im}C_V^2}$

$\text{Re}M_{12}^U$  and  $\text{Re}\Gamma_{12}^U$  are from the the above account of theoretics,  $\tan(d_U\pi)$  is playing the mainly role in above functions, it will decide the action of  $\text{Re}C_S^2, \text{Re}C_V^2, |\text{Im}C_S^2|$  and  $|\text{Im}C_V^2|$  with the  $d_U$  dimension. We can find that the value of the coefficient  $\text{Re}C_S^2$  and  $\text{Re}C_V^2$  in equation (24) will break sharp to infinity when the dimension  $d_U$  of  $\tan(d_U\pi)$  is 1/2 integral.  $|\text{Im}C_S^2|$  and  $|\text{Im}C_V^2|$  with the  $d_U$  also will break sharp to infinite when the dimension  $d_U$  of  $\tan(d_U\pi)$  is half integer and  $\bar{\varepsilon}$  is from the data of PDG[19]. And there is no  $\tan(d_U\pi)$  factor in equation (26). so the change of the  $\text{Re}C_S^2$  and  $\text{Re}C_V^2$  here are mild with the dimension  $d_U$ .

## E. Constrains between the dimensionless coefficients and fraction dimension

Thus we have got all equations of the relation of the  $C_S$ ,  $C_V$  and dimension " $d_U$ ". And then we will discuss constrains between the dimensionless coefficients and fraction dimension



TABLE I: The dimensionless coefficients  $C_S$ ,  $C_V$  with different scale dimension  $d_U$ . The first two column " $\sqrt{\text{Re}C_S^2}$ " and " $\sqrt{\text{Re}C_V^2}$ " are obtained from Eq. (24); the next two column " $\sqrt{\text{Re}C_S^2}$ " and " $\sqrt{\text{Re}C_V^2}$ " are obtained from Eq. (25) and the last two column " $\sqrt{\text{Im}C_S^2}$ " and " $\sqrt{\text{Im}C_V^2}$ " are obtained from Eq. (26). The energy scale  $\Lambda_U$  is chosen as  $\Lambda_U = 1$  TeV.

$d_U$	$\sqrt{\text{Re}C_S^2}$	$\sqrt{\text{Re}C_V^2}$	$\sqrt{\text{Re}C_S^2}$	$\sqrt{\text{Re}C_V^2}$	$\sqrt{\text{Im}C_S^2}$	$\sqrt{\text{Im}C_V^2}$
1.2	$4.3 \times 10^{-4}$	$2.1 \times 10^{-7}$	$9.3 \times 10^{-4}$	$4.8 \times 10^{-7}$	$6.1 \times 10^{-5}$	$3.2 \times 10^{-8}$
1.4	$4.6 \times 10^{-3}$	$2.4 \times 10^{-6}$	$4.8 \times 10^{-3}$	$2.5 \times 10^{-6}$	$6.5 \times 10^{-4}$	$3.4 \times 10^{-7}$
1.6	$2.8 \times 10^{-2}$	$1.5 \times 10^{-5}$	$3.0 \times 10^{-2}$	$1.6 \times 10^{-5}$	$4.1 \times 10^{-3}$	$2.1 \times 10^{-6}$
1.8	$9.5 \times 10^{-2}$	$4.9 \times 10^{-5}$	$2.1 \times 10^{-1}$	$1.1 \times 10^{-4}$	$1.4 \times 10^{-2}$	$7.0 \times 10^{-6}$
2.0	$1.3 \times 10^{-8}$	$6.6 \times 10^{-12}$	1.5	$7.8 \times 10^{-4}$	$1.8 \times 10^{-9}$	$9.4 \times 10^{-13}$
2.2	5.3	$2.7 \times 10^{-3}$	$1.1 \times 10$	$5.9 \times 10^{-3}$	$7.5 \times 10^{-1}$	$3.9 \times 10^{-3}$
2.4	$8.6 \times 10$	$4.5 \times 10^{-2}$	$9.1 \times 10$	$4.7 \times 10^{-1}$	$1.2 \times 10$	$6.4 \times 10^{-3}$
2.6	$7.0 \times 10^2$	$3.6 \times 10^{-1}$	$7.4 \times 10^2$	$3.8 \times 10^{-1}$	$1.0 \times 10^2$	$5.2 \times 10^{-2}$
2.8	$2.9 \times 10^3$	1.5	$6.2 \times 10^3$	3.2	$4.1 \times 10^2$	$2.1 \times 10^{-1}$
3.0	$5.6 \times 10^{-4}$	2.9	$5.4 \times 10^4$	$2.8 \times 10$	$7.9 \times 10^{-5}$	$4.1 \times 10^{-8}$

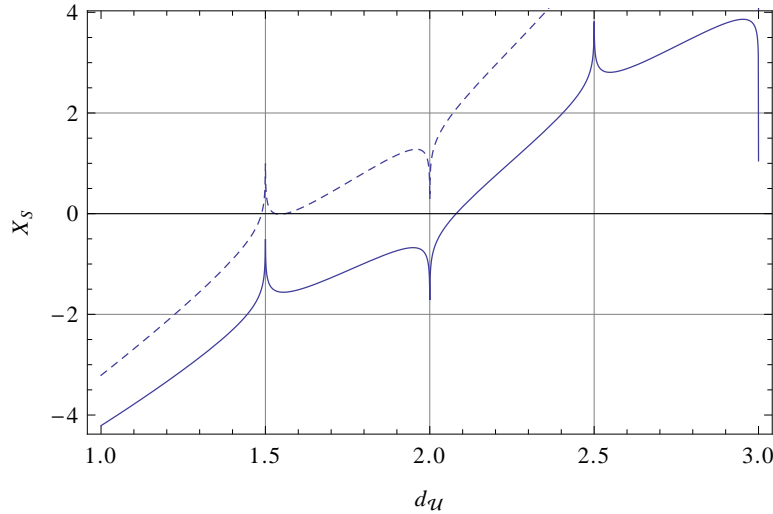


FIG. 2: The  $K^0 - \bar{K}^0$  mixing parameter  $X_S$  is from the first equation of formula (24) with scalar unparticle scale dimension ( $1.0 < d_U < 3.0$ ). The solid line is given for  $\Lambda_U = 1$  TeV and the dashed for  $\Lambda_U = 10$  TeV

" $d_U$ ". Let " $X_{(S,V)} = \text{Log}_{10} \sqrt{|\text{Re}C_{(S,V)}^2|}$ " and " $Y_{(S,V)} = \text{Log}_{10} \sqrt{|\text{Im}C_{(S,V)}^2|}$ " (these "V, S" is the sign of " scalar and vector " ), we will have 6 graphs (FIG.2-FIG.7) of these relation .

Account to the FIG.2-FIG.7 and TABLE 1, TABLE 2 of the  $K^0 - \bar{K}^0$  mixing we can find some points:

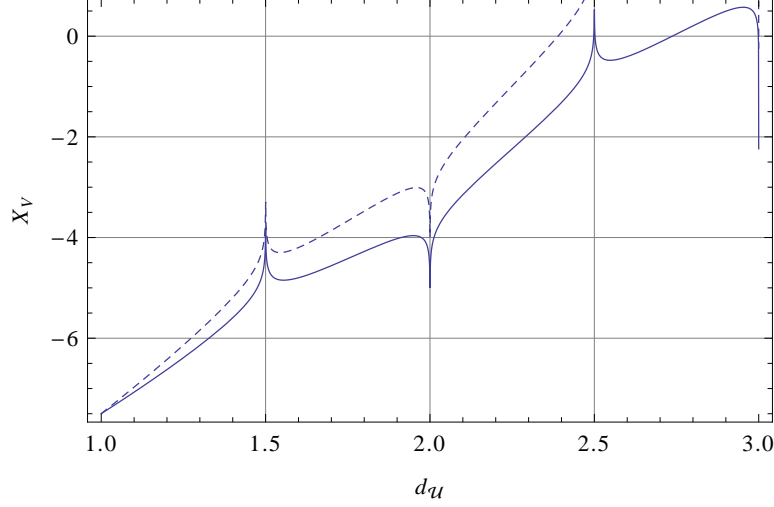


FIG. 3: The  $K^0 - \bar{K}^0$  mixing parameter  $X_S$  is from the second equation of formula (24) with vector unparticle scale dimension ( $1.0 < d_U < 3.0$ ). The solid line is given for  $\Lambda_U = 1 \text{ TeV}$  and the dashed for  $\Lambda_U = 10 \text{ TeV}$

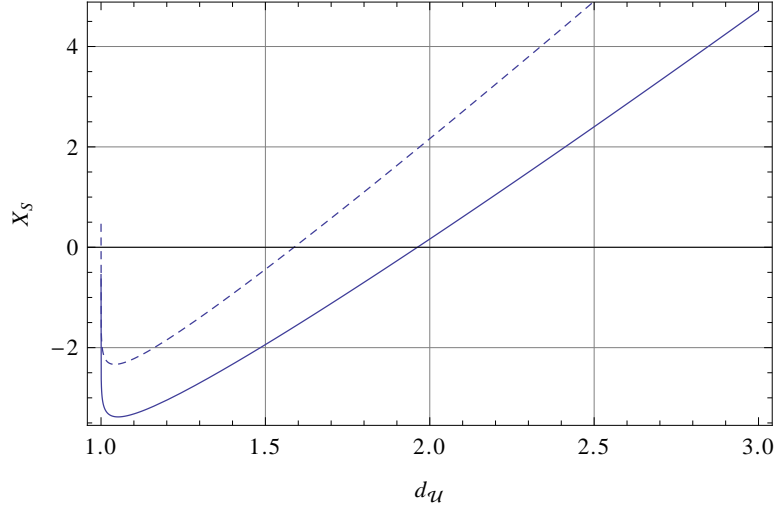


FIG. 4: The  $K^0 - \bar{K}^0$  mixing parameter  $X_S$  is from the first equation of formula(25) with scalar unparticle scale dimension ( $1.0 < d_U < 3.0$ ). The solid line is given for  $\Lambda_U = 1 \text{ TeV}$  and the dashed for  $\Lambda_U = 10 \text{ TeV}$

1. The FIG.2 and FIG.3 correspond to equation (24), the whole current is increasing rapidly, the change of the mixing parameter  $\text{Log}_{10}\sqrt{|ReC_S^2|}$  and  $\text{Log}_{10}\sqrt{|ReC_V^2|}$  are not holding smoothly when the scalar or vector unparticle scale dimension getting  $1.0 < d_U < 3.0$ , it will break sharply when the  $d_U$  are some given values. Especially, when the  $d_U\pi$  is  $\pi/2$  integral, the mixing parameter will break to some odd points and the curve will not be smooth. This is because of the term  $(\tan)$ , for the  $\tan$  function, the value will break to  $+\infty$  or  $-\infty$

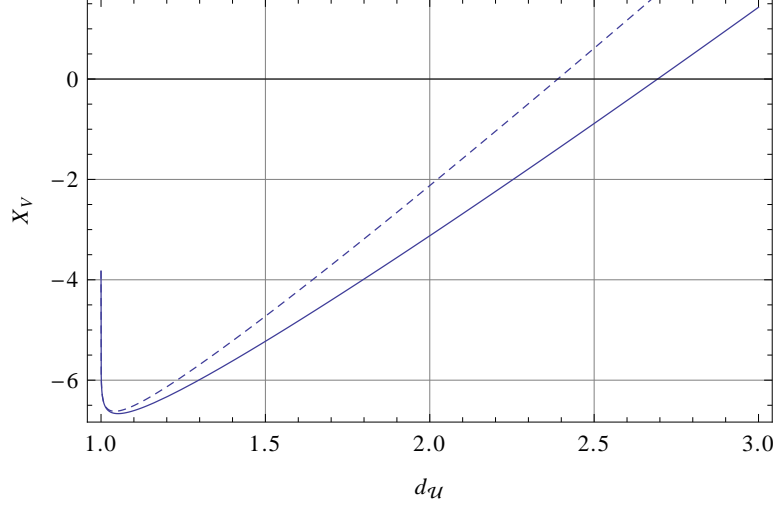


FIG. 5: The  $K^0 - \bar{K}^0$  mixing parameter  $X_V$  is from the second equation of formula (25) with vector unparticle scale dimension ( $1.0 < d_U < 3.0$ ). The solid line is given for  $\Lambda_U = 1 \text{ TeV}$  and the dashed for  $\Lambda_U = 10 \text{ TeV}$

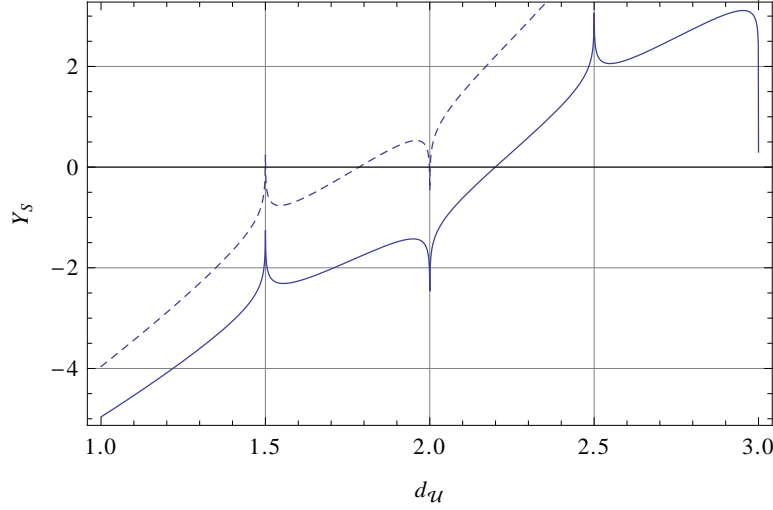


FIG. 6: The  $K^0 - \bar{K}^0$  mixing parameter  $Y_S$  is from the first equation of formula (26) with scalar unparticle scale dimension  $1.0 < d_U < 3.0$ . The solid line is given for  $\Lambda_U = 1 \text{ TeV}$  and the dashed for  $\Lambda_U = 10 \text{ TeV}$

when the variable is getting  $\pi/2$  integral. So the equation(24) will happen such movements. Another point is, we can make the parameter  $\Lambda_U = 1 \text{ TeV}$  and  $\Lambda_U = 10 \text{ TeV}$ , the different change of graph is just only on the parameter range of the function. Such parameter change will not influence the approximately profile, that can be seen clearly on the FIG.2 and FIG.3.

2. The different status are given on the FIG.4 and FIG.5 represent the dynamic properties of equation (25). The change of the mixing parameter  $\text{Log}_{10}\sqrt{|ReC_S^2|}$  and  $\text{Log}_{10}\sqrt{|ReC_V^2|}$  are

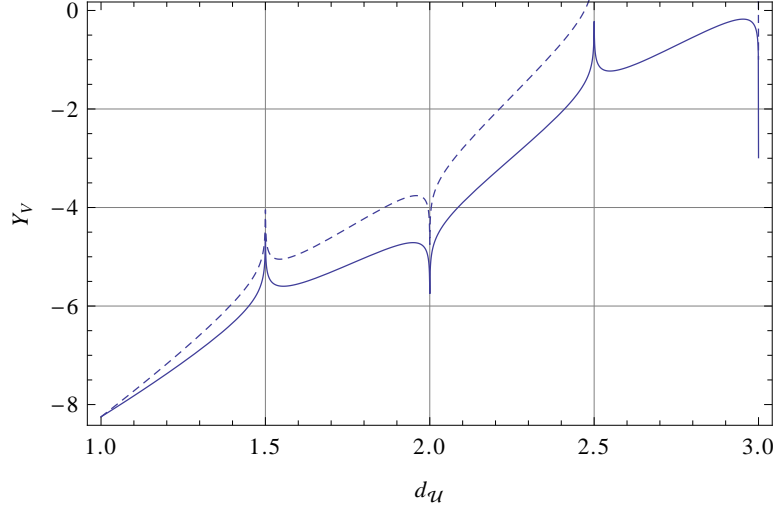


FIG. 7: The  $K^0 - \bar{K}^0$  mixing parameter  $Y_V$  is from the second equation of formula (26) with vector unparticle scale dimension  $1.0 < d_U < 3.0$ . The solid line is given for  $\Lambda_U = 1 \text{ TeV}$  and the dashed for  $\Lambda_U = 10 \text{ TeV}$

holding smooth and gradually increase rapidly with the dimension is getting  $1.0 < d_U < 3.0$ . The term  $(\tan)$  are not included, so the graph will not break in the range of the  $1.0 < d_U < 3.0$ .

3. The movement rules of the equation (26) are the same as those of equation(24). It also can be breaking to some oddity points and the change curve will not become smooth. See the FIG.6 and FIG.7, and contrast with the FIG.2 and FIG.3, the approximately profile are same in the  $(1.0 < d_U < 3.0)$ . It is also due to the  $(\tan)$  term in the equation(26), it also can happen the same circles that the value will break to the odd points when the variable is getting  $\pi/2$  integral. This will be very apparently and transparently when we notice that equation (24) and (26) have the same source, both from the first formula of equations (18) and (19), reflect the relations between  $C_S^2$  and  $C_V^2$  and the dimension  $d_U$ . In addition, from above discussions we can get the conclusion that the unparticle  $K^0 - \bar{K}^0$  mixing parameter  $C_S^2$  and  $C_V^2$  have the intense restriction with the change of the dimension  $d_U$ . So the  $K^0 - \bar{K}^0$  mixing may be present some new physics in some fractional dimension.

## V. CONCLUSIONS

In this paper, the new physics effects of the  $K^0 - \bar{K}^0$  mixing from scale invariant unparticle sectors have been explored. We consider sufficiently the contribution from the Standard Model part and the contribution from the new physics effects of the unparticle. They will contribute together to the experimental observations such as  $\Delta M_K^{exp}$ ,  $\Delta \Gamma^{exp}$  and  $|\bar{\epsilon}|$ . Overpass the account of the Standard Model part, we get the contribution of the new physics. In the

research of the  $K^0 - \bar{K}^0$  mixing, we find that the  $K^0 - \bar{K}^0$  mixing provides the most stringent constraint on the coupling mixing parameter of the scalar and vector unparticles with the dimension  $d_{\mathcal{U}}$ . The dependence of scale dimension  $d_{\mathcal{U}}$  shows that the mixing parameter is sensitive to the scale dimension, those sensitivity are presented on the FIG.2-FIG.7 and TABLE 1, and these obtained parameters' value may have important use on investigation of new physics.

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